Comparison of different measures for quantum discord under non-Markovian noise

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Abstract

Two geometric measures for quantum discord were recently proposed by Modi *et al.* [Phys. Rev. Lett. **104**, 080501 (2010)] and Dakić *et al.* [Phys. Rev. Lett. **105**, 190502 (2010)]. We study the similarities and differences for total quantum correlations of Bell-diagonal states using these two geometry-based quantum discord and the original quantum discord. We show that, under non-Markovian dephasing channels, quantum discord and one of the geometric measures stay constant for a finite amount of time, but not the other geometric measure. However, all the three measures share a common sudden change point. Our study on critical point of sudden transition might be useful for keeping long time total quantum correlations under decoherence.

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Entangled states cannot be prepared by local operations and classical communication [1, 2]. One may think that the exchange of classical information would not add any quantum correlation to the state. This is true for pure states but not for a general mixed state, because quantum correlations also exist in some mixed separable states and have played important roles in some quantum tasks, such as in deterministic quantum computation with one pure qubit [3]. To capture the total quantum correlations, a measure called quantum discord has been first proposed by Olliver and Zurek [4] and by Henderson and Vedral [5], and then widely studied [6–27].

Quantum discord is spoiled due to unavoidable interaction between the quantum system and the surrounding environment. The dynamics of quantum discord has been investigated under both Markovian [18–23] and non-Markovian [24] environments. Of particular interest is that, for a special class of quantum Bell-diagonal states, there exists sudden change of quantum discord under Markovian environment [19]. Moreover, the constant quantum discord under Markovian phase-damping channels was observed experimentally [20] and intensively studied theoretically [21, 22].

Quantum discord (capturing total quantum correlations) has also been explored from the aspect of geometry, where two measures were recently proposed based on, respectively, the relative entropy [25] and the square of Hilbert-Schmidt norm [26]. The present work concentrates on the comparison of these two geometric measures with the originally defined quantum discord. Since some interesting features, such as suddenly changing and constant total quantum correlations, have been discovered by the quantum discord with respect to Bell-diagonal states, we wonder if these features remain in the two geometric measures. We will focus on non-Markovian environments, from which we could obtain some insights into the protection of total quantum correlations from decoherence.

Quantum discord is defined as a measure of the discrepancy between two different quantum analogs of the classical mutual information [4, 5]. For a bipartite system ρ_{AB}, the quantum discord is given by

$$\mathcal{D}\left(\rho_{AB}\right) = \mathcal{I}\left(\rho_{AB}\right) - \mathcal{C}\left(\rho_{AB}\right), \tag{1}$$

where $\mathcal{I}\left(\rho_{AB}\right) = S\left(\rho_A\right) + S\left(\rho_B\right) - S\left(\rho_{AB}\right)$ is the quantum mutual information (also called total correlations [5]), with $S\left(\rho\right) = -\mathrm{Tr}(\rho \log_2 \rho)$ the von Neumann entropy of ρ and $\rho_{A(B)}$ the reduced density matrix of ρ_{AB} by tracing out $B(A)$. $\mathcal{C}\left(\rho_{AB}\right) = \max\limits_{B_k}\{S\left(\rho_A\right) - \sum_k q_k S\left(\rho_A^k\right)\}$

is considered as the classical correlation, where $\rho_A^k = \mathrm{Tr}_B \left\{ B_k \rho_{AB} B_k^\dagger \right\} / q_k$ is the resulting state after the measurement $\{B_k\}$ on B, and $q_k = \mathrm{Tr}_{AB} \left\{ B_k \rho_{AB} B_k^\dagger \right\}$. Note that quantum discord is not symmetric with respect to exchanging A and B, however, for the Bell-diagonal states under consideration, it is[26].

From the relative entropy perspective, the geometric measure $Q_{\mathcal{R}}$ quantifies how distinguishable a given state ρ is from the closest classical state υ [25],

$$Q_{\mathcal{R}}\left(\rho\right) = \min_{\upsilon \in \mathcal{G}} S\left(\rho || \upsilon\right), \tag{2}$$

where $\mathcal{G}$ is the set of classical states [28] and $S\left(\rho || \upsilon\right) = -\mathrm{Tr}(\rho \log_2 \upsilon) - S\left(\rho\right)$ is the relative entropy.

Another geometric measure $Q_{\mathcal{S}}$ is defined based on the fact that almost all quantum states have non-vanishing quantum discord [12, 26]. The distance between a given state ρ and the nearest zero-discord state ϱ is defined as [26],

$$Q_{\mathcal{S}}\left(\rho\right) = \min_{\varrho \in \Omega} ||\rho - \varrho||^2, \tag{3}$$

where Ω denotes the set of quantum states with zero-discord [28] and $||\rho - \varrho||^2 = \mathrm{Tr}(\rho - \varrho)^2$ is the square of Hilbert-Schmidt norm of Hermitian operators [26, 27]. For a two-qubit system $\rho_{AB} = (\mathbf{1} \otimes \mathbf{1} + \sum_{j=1}^{3} \alpha_j \sigma_j^A \otimes \mathbf{1} + \sum_{j=1}^{3} \beta_j \mathbf{1} \otimes \sigma_j^B + \sum_{j,k=1}^{3} \mathcal{M}_{jk} \sigma_j^A \otimes \sigma_k^B)/4$, with $\mathbf{1}$ and $\{\sigma_j\}$ being the identity and Pauli operators, Eq. (3) can be simplified as

$$Q_{\mathcal{S}}\left(\rho_{AB}\right) = \frac{1}{4}\left(||\vec{\alpha}||^2 + ||\mathcal{M}||^2 - \delta_{\max}\right), \tag{4}$$

where $\vec{\alpha} = \left(\alpha_1, \alpha_2, \alpha_3\right)^T$ is a column vector, $\mathcal{M} = (\mathcal{M}_{jk})$ is a matrix, and $\delta_{\max}$ is the largest eigenvalue of matrix $\vec{\alpha}\vec{\alpha}^T + \mathcal{M}\mathcal{M}^T$. Here the superscript T represents the transpose of vectors or matrices.

We start our analysis by considering two identical qubits A and B initially in a Bell-diagonal state with the density operator as

$$\rho_{AB}(0) = \frac{1}{4}\left(1 + \sum_{j=1}^{3} c_j \sigma_j^A \otimes \sigma_j^B\right) = \sum_{a,b=0,1} \lambda_{ab} |\chi_{ab}\rangle \langle \chi_{ab}|, \tag{5}$$

where the eigenstates are four Bell states $|\chi_{ab}\rangle = \big(|0, b\rangle + (-1)^a |1, 1 \oplus b\rangle\big)/\sqrt{2}$ with eigenvalues $\lambda_{ab} = \big(1 + (-1)^a c_1 - (-1)^{a+b} c_2 + (-1)^b c_3\big)/4$ (a,b=0,1)[9], and (c_1, c_2, c_3) are three parameters of the Bell-diagonal states. Considering $\lambda_{ab} \geq 0$, all Bell-diagonal states should be confined within a tetrahedron in three-dimensional space spanned by c_1, c_2, and c_3 [29].

In what follows, we consider the situation of the qubits under independent non-Markovian dephasing channels [30]. The dynamics of the qubits can be characterized by the Kraus operators $\{K_\mu(t)\}$: $\rho_{AB}(t) = \sum_\mu K_\mu(t)\rho_{AB}(0)K_\mu^\dagger(t)$, where the Kraus operators satisfy $\sum_\mu K_\mu^\dagger(t)K_\mu(t) = 1$. The Kraus operators for this non-Markovian model are given by $K_\mu(t) = \kappa_a(t)\otimes\kappa_b(t)$ $(a, b = 0, 1)$ where $\kappa_0(t) = \begin{pmatrix} \omega(t) & 0 \\ 0 & 1 \end{pmatrix}$ and $\kappa_1(t) = \begin{pmatrix} \sqrt{1-\omega^2(t)} & 0 \\ 0 & 0 \end{pmatrix}$, with $\omega(t) = \exp\big(-f(t)\big)$, $f(t) = \Gamma\big(t + (e^{-\gamma t} - 1)/\gamma\big)/2$, γ denotes the environmental noise bandwidth and Γ is the Markovian decay rate. Explicitly, the time evolution of the system can be expressed as $\rho_{AB}(t) = \sum_{a,b=0,1}\lambda_{ab}(t)|\chi_{ab}\rangle\langle\chi_{ab}|$, where $\lambda_{ab}(t) = \big(1 + (-1)^a c_1(t) - (-1)^{a+b}c_2(t) + (-1)^b c_3(t)\big)/4$, $c_1(t) = c_1(0)\omega^2(t)$, $c_2(t) = c_2(0)\omega^2(t)$, and $c_3(t) = c_3$ [$c_3(t)$ is constant during the evolution]. For simplicity, we denote in the following by $c_2(t) = \epsilon c_1(t)$, with $\epsilon = c_2(0)/c_1(0)$. In addition, the above results could return to the Markovian situation by setting $f(t) \to \Gamma t/2$ in the Markovian limit $\gamma \to \infty$.

According to Ref. [9], the classical correlation is calculated as

$$\begin{aligned} \mathcal{C}\big(\rho_{AB}(t)\big) &= \sum_{l=0,1} \frac{1+(-1)^l\mathbf{m}}{2}\log_2\big(1+(-1)^l\mathbf{m}\big), \\ &= 1 - H_{bin}\left(\frac{1+\mathbf{m}}{2}\right), \end{aligned} \tag{6}$$

where $\mathbf{m} = \max\{|c_1(t)|, |c_2(t)|, |c_3|\}$ and $H_{bin}(p) = -p\log_2 p - (1-p)\log_2(1-p)$ is the binary entropy [2]. In addition, the total correlation is $\mathcal{I}\big(\rho_{AB}(t)\big) = 2 + \sum_{a,b=0,1}\lambda_{ab}(t)\log_2\lambda_{ab}(t)$ [9], which, for the initial conditions $|c_1(0)| \geq |c_2(0)|, |c_3|$ and $\epsilon = -c_3$ [31], can be expressed as

$$\begin{aligned} \mathcal{I}\big(\rho_{AB}(t)\big) &= \sum_{\substack{l=0,1 \\ x=c_3,c_1(t)}} \frac{1+(-1)^l x}{2}\log_2\big(1+(-1)^l x\big), \\ &= 2 - H_{bin}\left(\frac{1+c_1(t)}{2}\right) - H_{bin}\left(\frac{1+c_3}{2}\right). \end{aligned} \tag{7}$$

Therefore, according to Eq. (1), the quantum discord is given by

$$\mathcal{D}\big(\rho_{AB}(t)\big) = \begin{cases} 1 - H_{bin}\left(\frac{1+c_3}{2}\right), & t \leq \tau, \\ 1 - H_{bin}\left(\frac{1+c_1(t)}{2}\right), & t > \tau, \end{cases} \tag{8}$$

where

$$\tau = \frac{1 + \eta\gamma + \mathcal{W}(-e^{-1-\eta\gamma})}{\gamma} \tag{9}$$

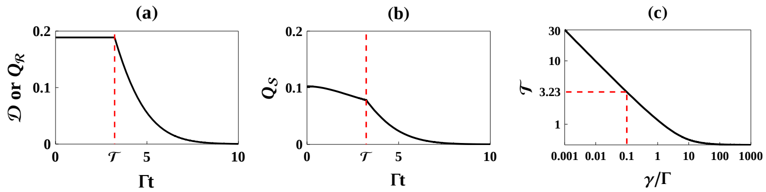

FIG. 1: Dynamics of total quantum correlations using $\mathcal{D}$ or $Q_{\mathcal{R}}$ in (a) and using $Q_{\mathcal{S}}$ in (b) as a function of Γt for Bell-diagonal state with the initial conditions $\big(c_1(0), c_2(0), c_3(0)\big) = (0.8, -0.4, 0.5)$ under non-Markovian dephasing channels with $\gamma/\Gamma = 0.1$. (c) is for the critical point $\mathcal{T} = \tau\Gamma$ with $|c_3/c_1(0)| = 5/8$ as a function of dimensionless scaled reservoir bandwidth γ/Γ ranging from non-Markovian regime to Markovian regime, where Γ is the Markovian decay rate.

is the critical point with $\eta = -\frac{\ln\left|\frac{c_3}{c_1(0)}\right|}{\Gamma}$ and $\mathcal{W}(\cdot)$ the Lambert $\mathcal{W}$ function. This is really an interesting phenomenon, since it seems to exist a 'decoherence-free' area of total quantum correlations when $t \leq \tau$ [20–22] [shown in Fig. 1(a)].

In Fig. 1(c), we have plotted the critical point $\mathcal{T}(= \Gamma\tau)$ as a function of dimensionless scaled reservoir noise bandwidth γ/Γ. We found that the critical point $\mathcal{T}$ grows with the decrease of the reservoir bandwidth. This implies that the non-Markovian behavior would prolong the quantum correlation under decoherence. When $\gamma \to \infty$ (Markovian limit), τ reduces to the cases studied in Ref. [21].

To calculate $Q_{\mathcal{R}}$, we denote the eigenvalues of Bell-diagonal states in a decreasing order by $\lambda_1(t) \geq \lambda_2(t) \geq \lambda_3(t) \geq \lambda_4(t)$. Therefore, the closest classical states of $\rho_{AB}(t)$ are of the form $\upsilon = \frac{\Lambda}{2}\big(|\lambda_1(t)\rangle\langle\lambda_1(t)| + |\lambda_2(t)\rangle\langle\lambda_2(t)|\big) + \frac{1-\Lambda}{2}\big(|\lambda_3(t)\rangle\langle\lambda_3(t)| + |\lambda_4(t)\rangle\langle\lambda_4(t)|\big)$ [32], with $\Lambda = \lambda_1(t) + \lambda_2(t)$. So the relative entropy based quantum discord is given by

$$Q_{\mathcal{R}}\big(\rho_{AB}(t)\big) = \sum_{a,b=0,1} \lambda_{ab}(t)\log_2\lambda_{ab}(t) + H_{bin}(\Lambda) + 1. \tag{10}$$

We may find $H_{bin}(\Lambda) = H_{bin}\left(\frac{1+\mathbf{m}}{2}\right)$ in both Markovian and non-Markovian regimes, which implies that $Q_{\mathcal{R}}$ and $\mathcal{D}$ are equivalent for Bell-diagonal states.

On the other hand, for the Bell-diagonal states under non-Markovian dephasing channels, the geometric measure of quantum discord based on the square of Hilbert-Schmidt norm can

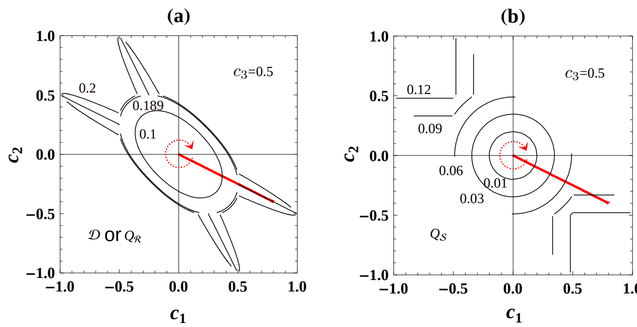

FIG. 2: Contour maps for total quantum correlations of Bell-diagonal states using the definitions of (a) $\mathcal{D}$ or $Q_{\mathcal{R}}$, and (b) $Q_{\mathcal{S}}$, respectively. The red straight lines $[c_2(t) = \epsilon c_1(t)$ with $\epsilon = c_2(0)/c_1(0)]$ represent the trajectories of the Bell-diagonal state under non-Markovian dephasing channels with the initial conditions $\big(c_1(0), c_2(0), c_3(0)\big) = (0.8, -0.4, 0.5)$. The red-dotted circular arrows represent the possible distribution of other lines through the origin of coordinate, corresponding to trajectories of Bell-diagonal states with other possible initial conditions.

be obtained exactly as follows [26]

$$\begin{aligned} Q_{\mathcal{S}}\big(\rho_{AB}(t)\big) \;=\; & \frac{1}{4}\big(c_1^2(t) + c_2^2(t) + c_3^2 \\ & - \max\{c_1^2(t), c_2^2(t), c_3^2\}\big). \end{aligned} \tag{11}$$

Clearly, with the initial conditions $|c_1(0)| \geq |c_2(0)|, |c_3|$ and $\epsilon = -c_3$, $c_2^2(t)$ will not be larger than $c_1^2(t)$ and $Q_{\mathcal{S}}$ is strongly dependent on the relation between $|c_1(t)|$ and $|c_3|$. Therefore, the geometric quantum discord in such a case can be written as

$$Q_{\mathcal{S}}\big(\rho_{AB}(t)\big) = \begin{cases} \big(c_2^2(t) + c_3^2\big)/4, & t \leq \tau, \\ \big(c_1^2(t) + c_2^2(t)\big)/4, & t > \tau, \end{cases} \tag{12}$$

which involves no constant total quantum correlations, as shown in Fig. 1(b).

This phenomenon is quite different from the cases measured by $\mathcal{D}$ or $Q_{\mathcal{R}}$. Although they share a common critical point τ, the original 'decoherence-free' area of total quantum correlations discovered by $\mathcal{D}$ or $Q_{\mathcal{R}}$ [20–22] does not appear in the measure of $Q_{\mathcal{S}}$.

To be more clarified, we have plotted in Fig. 2 the contour maps of quantum discord in a two-dimensional coordinate space with $c_3 = 0.5$. Recalling $c_2(t) = \epsilon c_1(t)$ with $\epsilon = \frac{c_2(0)}{c_1(0)}$, the possible trajectories under the non-Markovian dephasing channels should be the straight lines crossing the origin of coordinate (the red-dotted circular arrows represent the distribution of line's slope, i.e., different initial conditions). For a special case of $c_1(0) = 0.8$

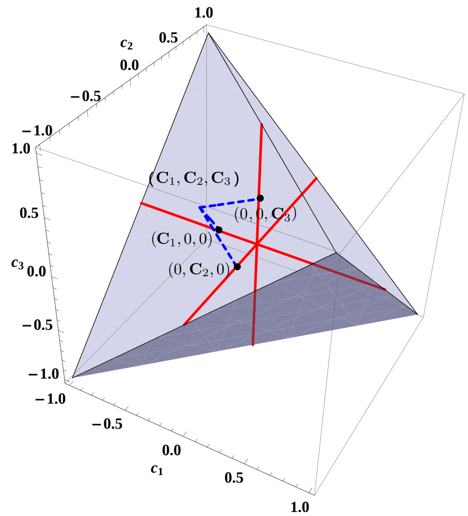

FIG. 3: The set of Bell-diagonal states with three parameters (c_1, c_2, c_3). The red lines represent zero-discord states. The blue-dashed lines connect to the possible nearest zero-discord states for a given Bell-diagonal state $(\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3)$.

and $c_2(0) = -0.4$, the trajectory of the Bell-diagonal states under decoherence is depicted as the red lines in Fig. 2. Clearly, the red line coincides with the straight contour line in Fig. 2(a), which means the quantum discord will not be spoiled in this 'decoherence-free' area [20–22]. In addition, there are three other 'decoherence-free' areas (See the black straight contour lines, one for $\epsilon = -c_3$ and the other two for $\epsilon = -1/c_3$[31]) as depicted in Fig. 2(a). For other values of ϵ (initial conditions), the red line will always cross the contour lines and no constant quantum discord will take place. However, in Fig. 2(b), the red straight lines always definitely go through the contour lines, which is a direct illustration of no constant quantum discord by $Q_{\mathcal{S}}$.

Since all the three quantities are to measure the total quantum correlations, we wonder why $Q_{\mathcal{S}}$ is incompatible with $\mathcal{D}$ and $Q_{\mathcal{R}}$ in describing the dynamics of Bell-diagonal states under non-Markovian dephasing channels? As both $Q_{\mathcal{R}}$ and $Q_{\mathcal{S}}$ are defined from geometric perspective, we guess the imcompatibility is from the fact that the nearest zero-discord state, belonging to an arbitrary Bell-diagonal state and measured by the square of Hilbert-Schmidt norm, is different from the closest classical state quantified by the relative entropy.

The guess is checked below. As demonstrated in Ref. [26], the set of zero-discord in a three-dimensional space spanned by (c_1, c_2, c_3) includes three mutually perpendicular lines $\{c_j \in [-1, 1] \mid c_k = 0, k \neq j \ (j = 1, 2, 3)\}$ (red lines in Fig. 3) and the zero-discord states can

be written as $\Omega = (1 + c_j\sigma_j \otimes \sigma_j)/4$ $(j = 1, 2, 3)$. For an arbitrarily given Bell-diagonal state ρ denoted by $(\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3)$, the distance to the zero-discord states measured by the square of Hilbert-Schmidt norm can be calculated as

$$\begin{aligned} ||\rho - \Omega||^2 &= Tr\{\Big(\frac{(\mathbf{C}_j - c_j)\sigma_j \otimes \sigma_j + \mathbf{C}_k\sigma_k \otimes \sigma_k + \mathbf{C}_l\sigma_l \otimes \sigma_l}{4}\Big)^2\} \\ &= \frac{(\mathbf{C}_j - c_j)^2 + \mathbf{C}_k^2 + \mathbf{C}_l^2}{4}(j \neq k \neq l). \end{aligned} \tag{13}$$

Clearly, $||\rho - \Omega||^2$ reaches the minimum only when $\mathbf{C}_j - c_j = 0$. Therefore, the possible nearest zero-discord states should be from $(\mathbf{C}_1, 0, 0)$, $(0, \mathbf{C}_2, 0)$, and $(0, 0, \mathbf{C}_3)$, dependent on the magnitude among $|\mathbf{C}_1|$, $|\mathbf{C}_2|$, and $|\mathbf{C}_3|$. For example, when $\mathbf{C}_1 > \mathbf{C}_2 > \mathbf{C}_3 > 0$, the nearest zero-discord state is $(\mathbf{C}_1, 0, 0)$.

On the other hand, as λ_{00} and λ_{01} are larger than λ_{10} and λ_{11} in the case of $\mathbf{C}_1 > \mathbf{C}_2 > \mathbf{C}_3 > 0$ (in other cases with arbitrary ordinal relation for $|\mathbf{C}_1|, |\mathbf{C}_2|$, and $|\mathbf{C}_3|$, the proof is similar), we have $\Lambda = \lambda_{00} + \lambda_{01} = (1 + \mathbf{C}_1)/2$. Recalling the requirements for the closest classical state measured by the relative entropy [32], we can obtain $\lambda_{00} = \lambda_{01} = \Lambda/2$ and $\lambda_{10} = \lambda_{11} = (1 - \Lambda)/2$, i.e., $\mathbf{C}_2 = \mathbf{C}_3 = 0$. Therefore, the closest classical state measured by the relative entropy is also $(\mathbf{C}_1, 0, 0)$, which coincides with the nearest zero-discord state measured by the square of Hilbert-Schmidt norm.

Since for an arbitrary Bell-diagonal state, the nearest zero-discord state measured by $Q_{\mathcal{S}}$ is just the closest classical state quantified by $Q_{\mathcal{R}}$, the discrepancy we discovered in this work must be resulted from the intrinsic nature of the square of Hilbert-Schmidt norm and the relative entropy.

To summarize, we have investigated quantum discord of Bell-diagonal states under decoherence by three different definitions. The differences and similarities by using the three measures have been presented and discussed. The study of critical point under non-Markovian environment might be helpful for prolonging total quantum correlations under decoherence.

Finally, it would be really interesting to further explore the 'decoherence-free' area of total quantum correlations, which appears in the originally defined quantum discord $\mathcal{D}$ and relative entropy based $Q_{\mathcal{R}}$, but disappears from the geometric perspective $Q_{\mathcal{S}}$. It may lead to a more fundamental quantum information problem, that is, which measure, $\mathcal{D}$ $(Q_{\mathcal{R}})$ or $Q_{\mathcal{S}}$, is more accurate to characterize total quantum correlations?

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